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**THE METHOD SIMPLIFYING INVERSE
LAPLACE TRANSFORMATION AT
OSCILLATORY PROCESSES RESEARCHES.
THE "AMPLITUDE, PHASE, FREQUENCY" PROBLEM
IN RADIOELECTRONICS AND ITS SOLUTION**

Tutorial

This book is recommended by Siberian regional department
of teaching methodological association education in the field
of energetics and electrotechniques as educational textbook
for interuniversity practice

Zolotarev I.D.

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The research method of transient processes in oscillatory systems
is stated. It permits essential simplification of the most difficult op-
eration in finding solution of a system differential equation – inverse
Laplace transformation. It is shown that complex signal provides cor-
rect definition of an envelope and phase of a real signal using this
method. The obviousness of obtained solutions is achieved by engag-
ing the spectral method. The examples of transient processes calcula-
tion in developing radioelectronic devices are given.

This tutorial is intended for students, post-graduate students, en-
gineers, scientific employees of both radio and electrotechnical speci-
alities and also specialists in the field of measuring and automation
technology while researching dynamics of oscillating systems.

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INTRODUCTION

At development of radioelectronic devices for different purposes an engineer frequently has to solve a researching problem of impulse radiosignals passing through linear circuits. For solving the task in the time domain the operational calculus based on integral Laplace transformations is widely used. When considering the task in the frequency domain the spectral method based on integral Fourier transformations is applied. Both these research approaches are tightly interlinked among themselves and sometimes are considered as a uniform method (the method of Fourier transformation).

When finding a response of a radioelectronic device (RED) to an impulse energization applying the operational calculus the most difficult operation is the inverse Laplace transformation (ILT) execution [1]. The difficulty of ILT especially increases for important radioelectronic applications when a radioimpulse signal affects RED and if selective filters are included in a signal tract of RED (oscillating system). It is stipulated by the fact that for radioimpulse signals and such realizations of RED the imaging function (IF) of the researching system response for the input disturbance has complex conjugate poles (CCP) pairs. In these cases even for rather simple IF the difficulty and awkwardness of conversions when turning from the images space to the originals one essentially raise in comparison with finding solutions for real poles IF [2]. In the meantime the existing tendency of extreme increasing of information processing speed in radiosystems requires to develop RED working in the dynamic mode when the conversions of a signal, taking off and processing its informative parameter are executed not after the termination of transient processes (TP) in the output of the informative channel but during these processes. Generally because of inevitable TP presence at energization of a radioelectronic system by an impulse signal the form of it is distorted. The specified distortions corrupt the informative parameter of a signal (originate definite dynamic errors in system functioning). The researching TP in a system for the purpose of minimization of an error imported to the signal informative parameter by transient processes is one of necessary development stages for modern RED operating in the dynamic mode. Therefore the problem of development of methods simplifying researches of

transient processes in electronic devices always attracts a serious attention of specialists [1–6].

The most prevalent at researches of transient processes in radiosystems is the method of slowly varying envelopes (SVE) designed by S. I. Evtyanov. In this method essential decreasing of difficulty in solving linear differential equations (DE) while researching TP in oscillating systems is achieved applying some particular simplifying assumptions (asymptotic method of the small parameter). In this case the initial DE communicating the response of the linear system and the energizing radiosignal are converted into truncated symbolical equations regarding to SVE [2]. The more narrow-band signals and systems are studied the more precise solutions are obtained using the SVE method. As a measure of band narrowity of radiosignals and systems the ratios $\mu = \Delta\omega_s / \omega_c$ and $\varepsilon = 2\Delta\omega_c / \omega_r$, where $\Delta\omega_s$ – the width of a radiosignal spectrum, ω_c – the filling high-frequency (HF), $2\Delta\omega_c$ – the bandwidth of oscillating system, ω_r – resonant system frequency are usually considered. For narrow-band signals and systems we have the small parameters μ and ε ($\mu \ll 1$, $\varepsilon \ll 1$). For wide-band and ultra wide-band systems these parameters are comparable to the unit.

Although the method of S. I. Evtyanov allows to simplify essentially the difficulties in finding an enough precise solution for an envelope of a signal in the output of a radiosystem it does not provide the authentic description of thin (phase) structure of an output radiosignal. Considering the greatest possibilities and advantages of phase information radiosystems operating in the dynamic mode [7] we notice that the specified disadvantage of the SVE method is rather essential.

Designed in [5–10] method simplifying inverse Laplace transformation ensures the same reducing of difficulties in solution obtaining as the SVE method. However when using the method [5–10] the precise (accurate to phase) solution for the radiosignal in the output of an investigated radiosystem is obtained. Thus it is not necessary to introduce simplifying assumptions which are peculiar to asymptotic methods including the SVE one.

Apart from great simplification of a solution determination by the method [5–10], application of it for the important cases of oscillatory processes researches allows to obtain a system response definition as a complex signal (CS). It facilitates dynamic modes in radiosystems

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